# ME 245 : ENGINEERING MECHANICS AND THEORY OF MACHINES

LECTURE: GEAR AND GEAR TRAINS

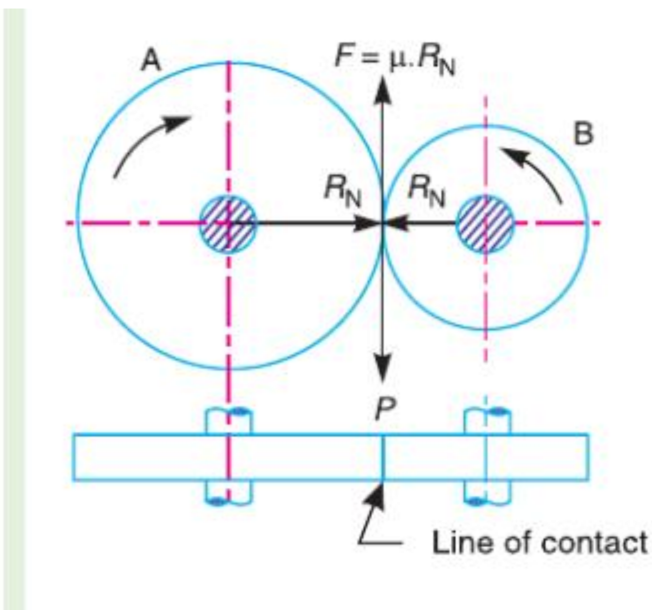
MD. TANVER HOSSAIN

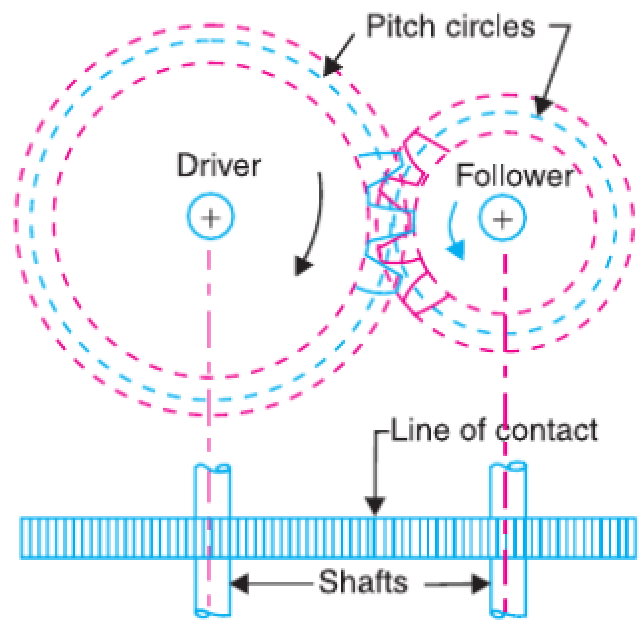
DEPARTMENT OF MECHANICAL ENGINEERING, BUET

http://tantusher.buet.ac.bd

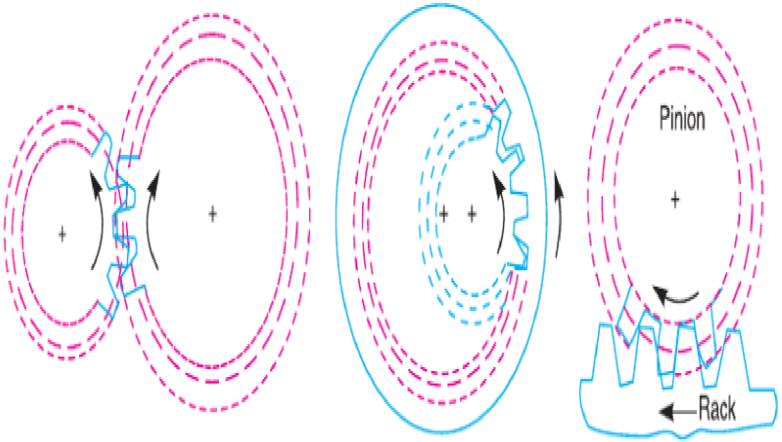
# GEARING

- The motion and power transmitted by gears is kinematically equivalent to that transmitted by friction wheels or discs.



# GEARING

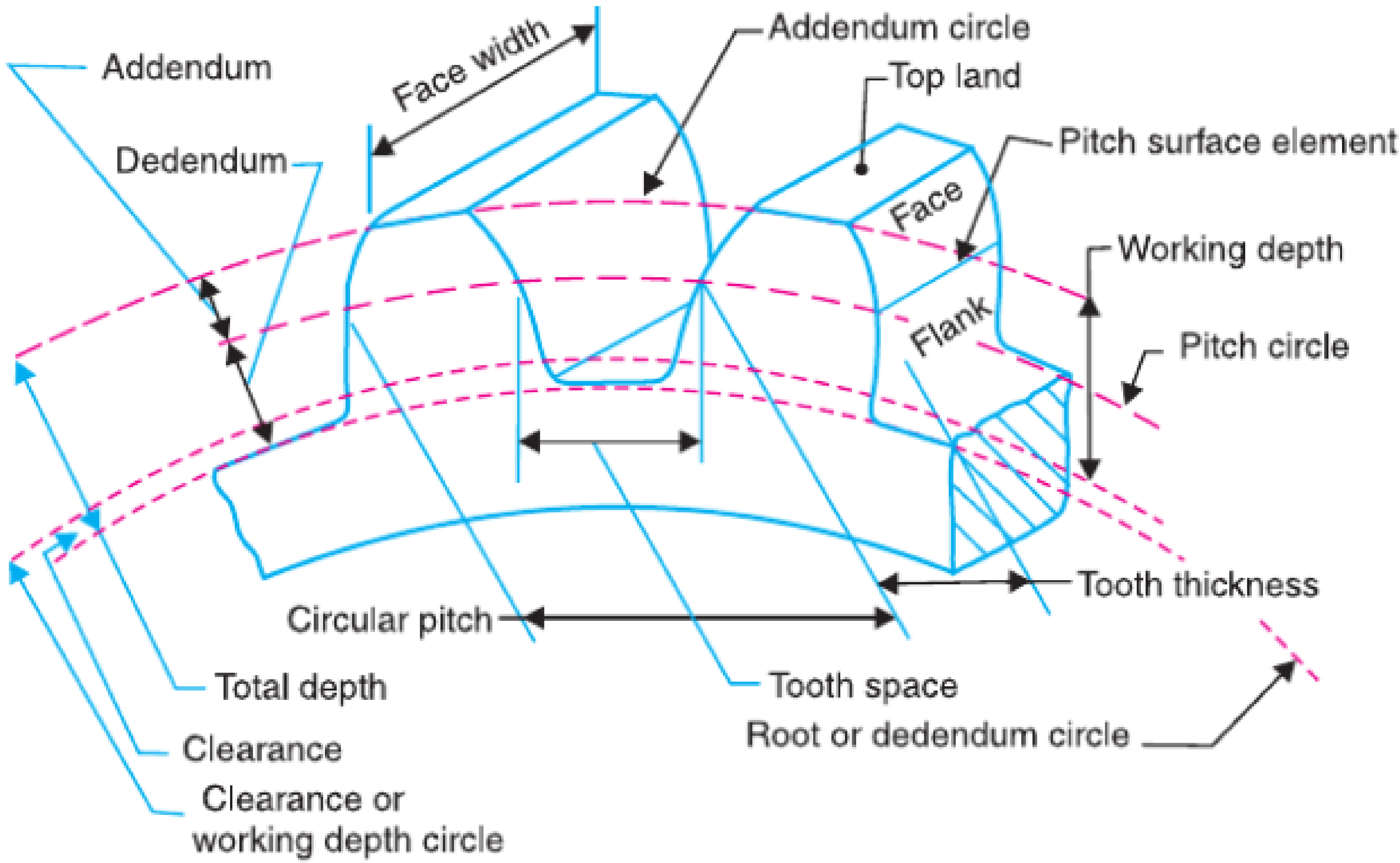




(*a*) External gearing. (*b*) Internal gearing.

**Fig. 12.3**

**Fig. 12.4.** Rack and pinion.

# GEARING


Addendum
Dedendum
Face width
Addendum circle
Top land
Pitch surface element
Face
Working depth
Flank
Pitch circle
Tooth thickness
Circular pitch
Total depth
Tooth space
Root or dedendum circle
Clearance
Clearance or
working depth circle

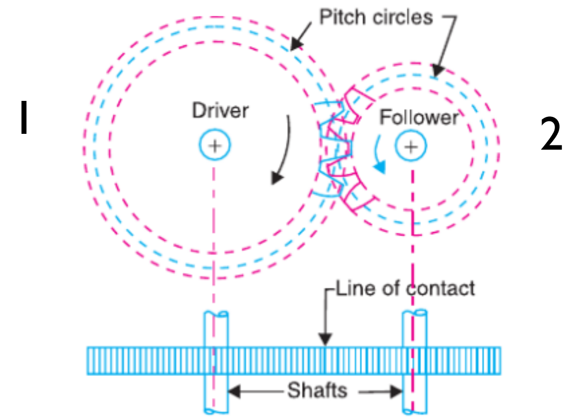
**10.** ***Circular pitch***. It is the distance measured on the circumference of the pitch circle from a point of one tooth to the corresponding point on the next tooth. It is usually denoted by $p_c$. Mathematically,

$$\text{Circular pitch,} \quad p_c = \pi D/T$$

$D$ = Diameter of the pitch circle, and

$T$ = Number of teeth on the wheel.

$$p_c = \frac{\pi D_1}{T_1} = \frac{\pi D_2}{T_2} \quad \text{or} \quad \frac{D_1}{D_2} = \frac{T_1}{T_2}$$




**3.** If $D_1$ and $D_2$ are pitch circle diameters of wheels 1 and 2 having teeth $T_1$ and $T_2$ respectively, then velocity ratio,

$$\frac{\omega_1}{\omega_2} = \frac{O_2P}{O_1P} = \frac{D_2}{D_1} = \frac{T_2}{T_1}$$

**11. *Diametral pitch.*** It is the ratio of number of teeth to the pitch circle diameter in millimetres. It is denoted by $p_d$ . Mathematically,

$$\text{Diametral pitch,} \quad p_d = \frac{T}{D} = \frac{\pi}{p_c} \qquad \ldots \left( \because\ p_c = \frac{\pi D}{T} \right)$$

where $T$ = Number of teeth, and

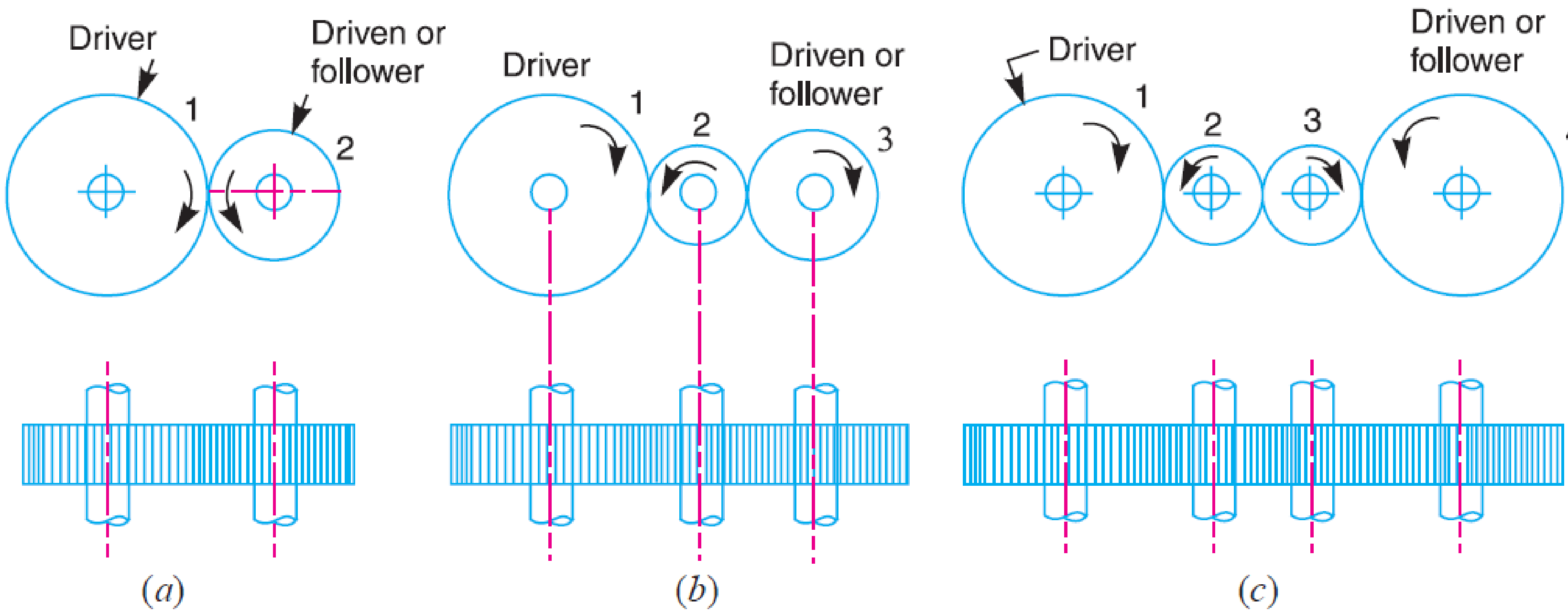
$D$ = Pitch circle diameter.

**12. *Module.*** It is the ratio of the pitch circle diameter in millimeters to the number of teeth. It is usually denoted by $m$. Mathematically,

$$\text{Module,} \quad m = D/T$$

# GEAR TRAIN

- Sometimes, two or more gears are made to mesh with each other to transmit power from one shaft to another. Such a combination is called ***gear train or train of toothed wheels.***
- Following are the different types of gear trains, depending upon the arrangement of wheels :**1. Simple gear train, 2. Compound gear train, 3.** Reverted gear train, and **4. Epicyclic gear train.**




$N_1$ = Speed of gear 1(or driver) in r.p.m.,

$N_2$ = Speed of gear 2 (or driven or follower) in r.p.m.,

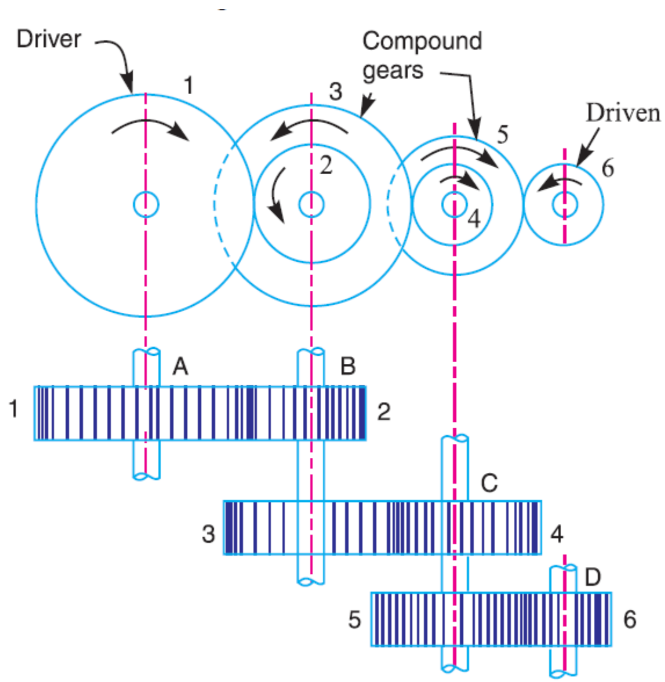
$T_1$ = Number of teeth on gear 1, and

$T_2$ = Number of teeth on gear 2.

$$\text{Speed ratio} = \frac{N_1}{N_2} = \frac{T_2}{T_1}$$

$$\frac{N_1}{N_2} = \frac{d_2}{d_1} = \frac{T_2}{T_1}$$

# COMPOUND GEAR TRAIN




$$\text{Speed ratio} = \frac{\text{Speed of the first driver}}{\text{Speed of the last driven or follower}}$$

$$= \frac{\text{Product of the number of teeth on the drivens}}{\text{Product of the number of teeth on the drivers}}$$

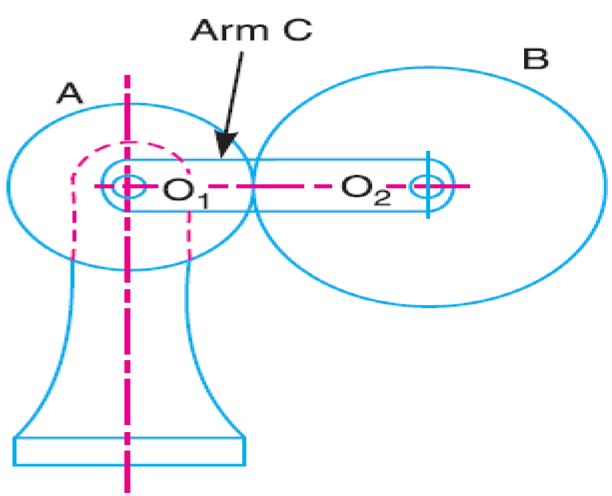
A compound train consists of six gears. The number of teeth on the gears are as follows :

| Gear : | $A$ | $B$ | $C$ | $D$ | $E$ | $F$ |
|---|---|---|---|---|---|---|
| No. of teeth : | 60 | 40 | 50 | 25 | 30 | 24 |

The gears $B$ and $C$ are on one shaft while the gears $D$ and $E$ are on another shaft. The gear $A$ drives gear $B$, gear $C$ drives gear $D$ and gear $E$ drives gear $F$. If the gear $A$ transmits 1.5 kW at 100 r.p.m. and the gear train has an efficiency of 80 per cent, find the torque on gear $F$.

# EPICYCLIC GEAR TRAIN

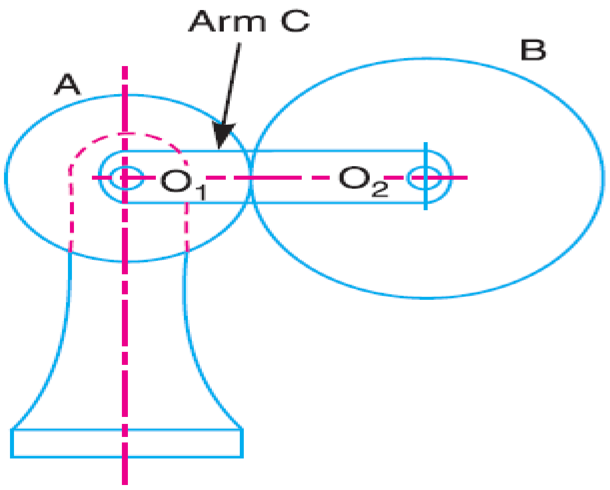
- A gear *A and the arm C have a common axis at $O_1$ about which they can rotate. The* gear *B meshes with gear A and h as its axis on the arm at $O_2$, about which the gear B can rotate*. If the arm is fixed, the gear train is simple and gear *A can drive gear B* or ***vice-versa, but if gear A is fixed and the arm is rotated about*** the axis of gear *A (i.e. $O_1$), then the gear B is forced to rotate* ***upon and around gear A. Such a motion is called epicyclic and*** the gear trains arranged in such a manner that one or more of their members move upon and around another member are known as ***epicyclic gear trains (epi. means upon and cyclic*** means around). The epicyclic gear trains may be ***simple or compound.***



Let $T_A$ = Number of teeth on gear $A$, and

$T_B$ = Number of teeth on gear $B$.

First of all, let us suppose that the arm is fixed.



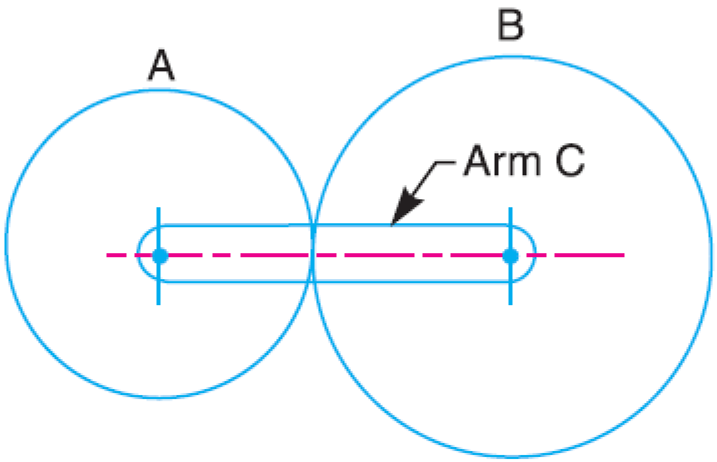

Therefore the axes of both the gears are also fixed relative to each other. When the gear *A makes one* revolution anticlockwise, the gear *B will* make ***$T_A$ / $T_B$ revolutions, clockwise.**

Assuming the anticlockwise rotation as positive and clockwise as negative, we may say that when gear *A makes* + *1* revolution, then the gear *B will make* (– $T_A$ / $T_B$) *revolutions.*

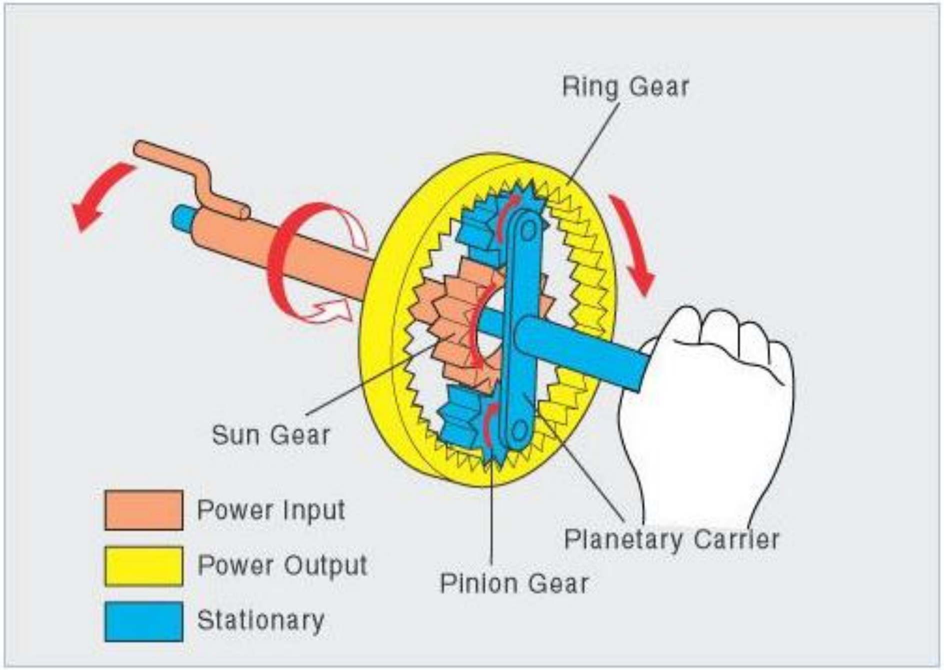
| Step No. | Conditions of motion | Revolutions of elements | | |
|---|---|---|---|---|
| | | Arm C | Gear A | Gear B |
| 1. | Arm fixed-gear $A$ rotates through + 1 revolution *i.e.* 1 rev. anticlockwise | 0 | + 1 | $-\frac{T_A}{T_B}$ |
| 2. | Arm fixed-gear $A$ rotates through + $x$ revolutions | 0 | $+x$ | $-x \times \frac{T_A}{T_B}$ |
| 3. | Add $+y$ revolutions to all elements | $+y$ | $+y$ | $+y$ |
| 4. | Total motion | $+y$ | $x+y$ | $y - x \times \frac{T_A}{T_B}$ |

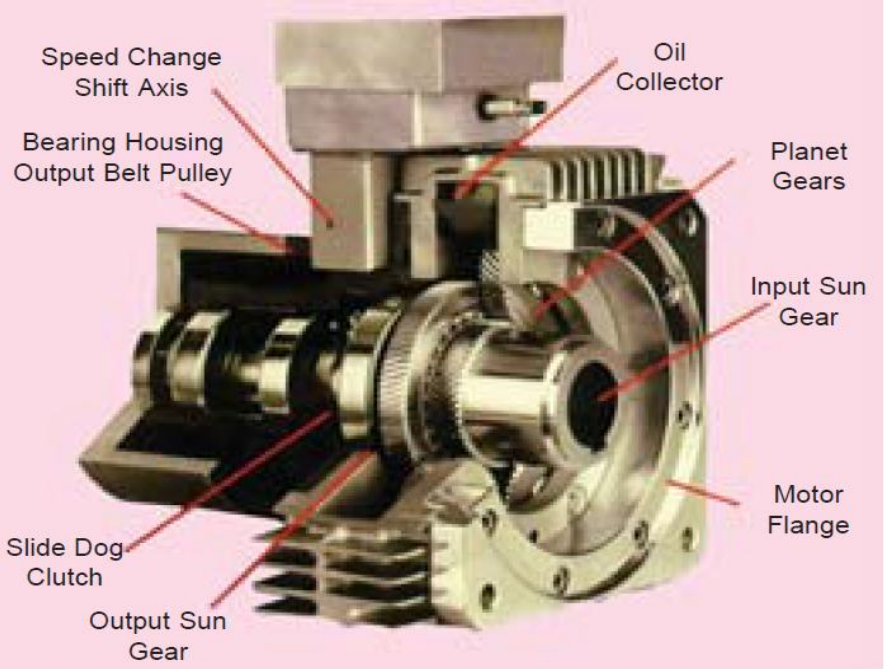
*In an epicyclic gear train, an arm carries two gears A and B having 36 and 45 teeth respectively. If the arm rotates at 150 r.p.m. in the anticlockwise direction about the centre of the gear A which is fixed, determine the speed of gear B. If the gear A instead of being fixed, makes 300 r.p.m. in the clockwise direction, what will be the speed of gear B ?*

| *Step No.* | *Conditions of motion* | *Revolutions of elements* | | |
|---|---|---|---|---|
| | | *Arm C* | *Gear A* | *Gear B* |
| 1. | Arm fixed-gear *A* rotates through + 1 revolution *i.e.* 1 rev. anticlockwise | 0 | + 1 | $-\frac{T_A}{T_B}$ |
| 2. | Arm fixed-gear *A* rotates through + *x* revolutions | 0 | $+x$ | $-x \times \frac{T_A}{T_B}$ |
| 3. | Add + *y* revolutions to all elements | $+y$ | $+y$ | $+y$ |
| 4. | Total motion | $+y$ | $x+y$ | $y - x \times \frac{T_A}{T_B}$ |

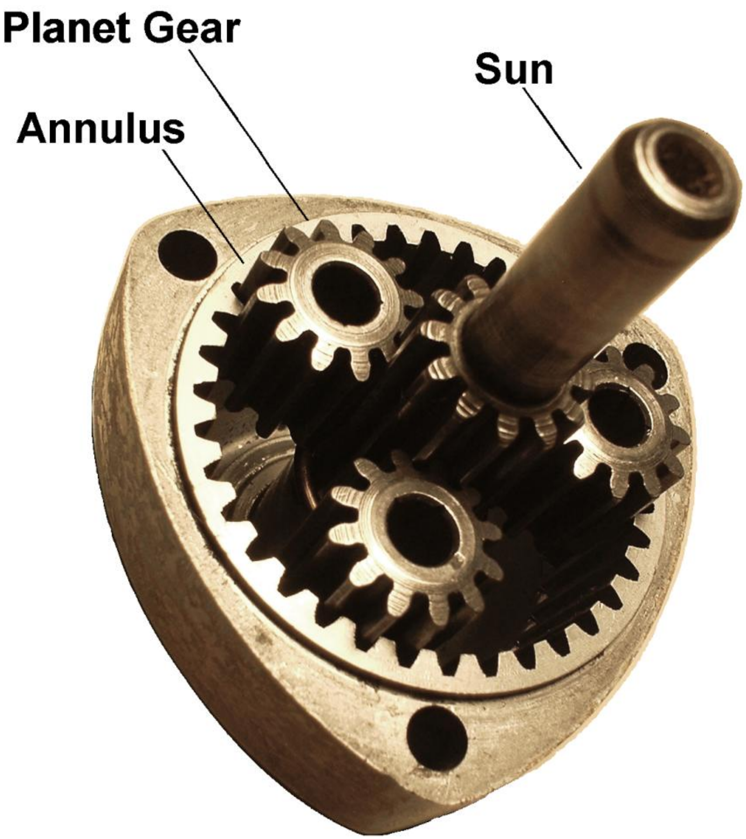


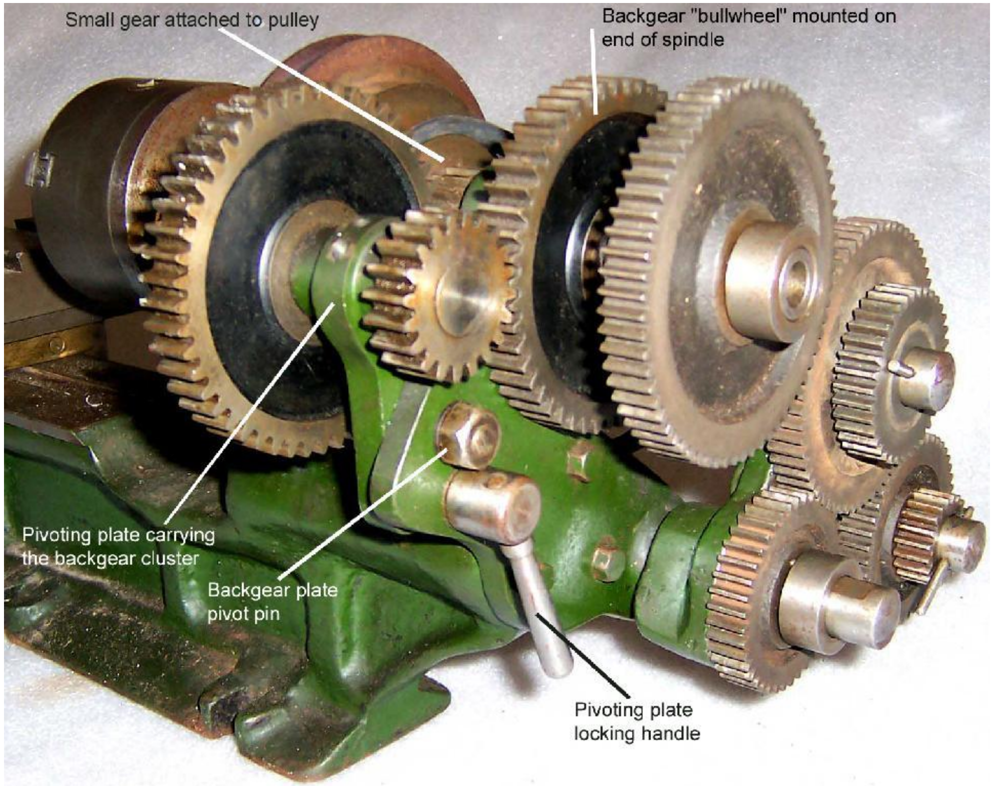
# Compound Epicyclic Gear Train—Sun and Planet Gear

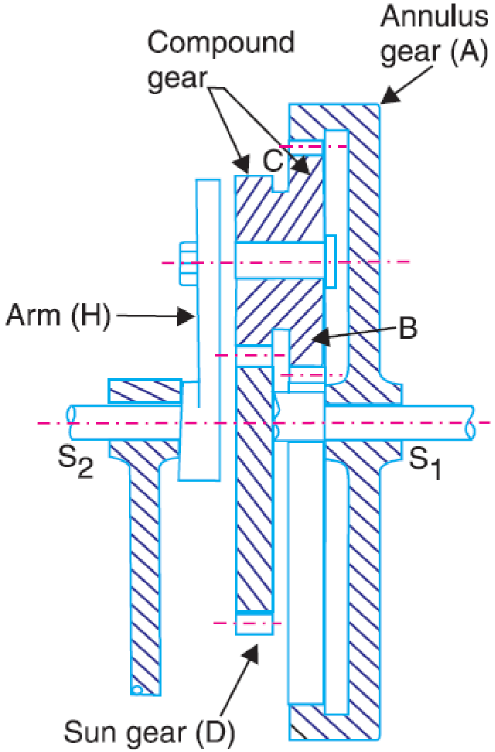


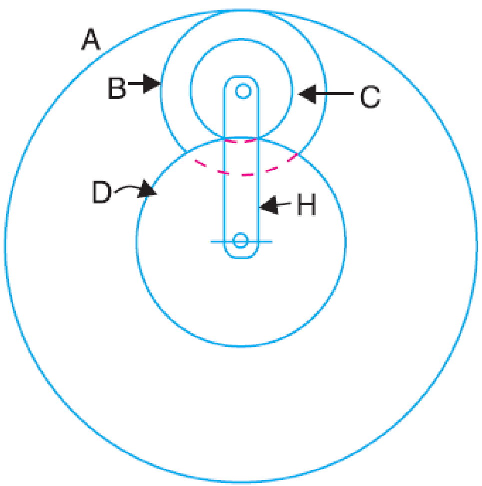




Sun and Planet gears.


Planet Gear
Sun
Annulus

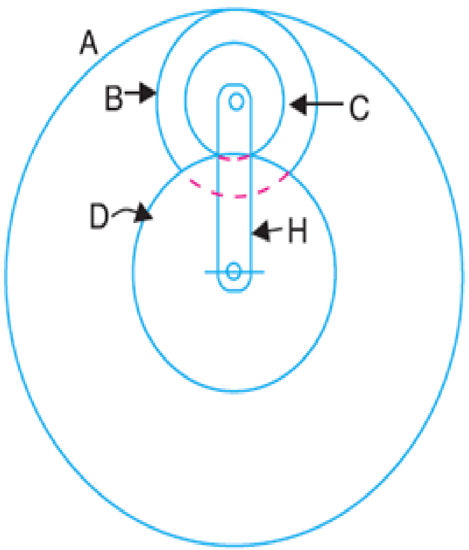

Small gear attached to pulley
Backgear "bullwheel" mounted on end of spindle
Pivoting plate carrying the backgear cluster
Backgear plate pivot pin
Pivoting plate locking handle



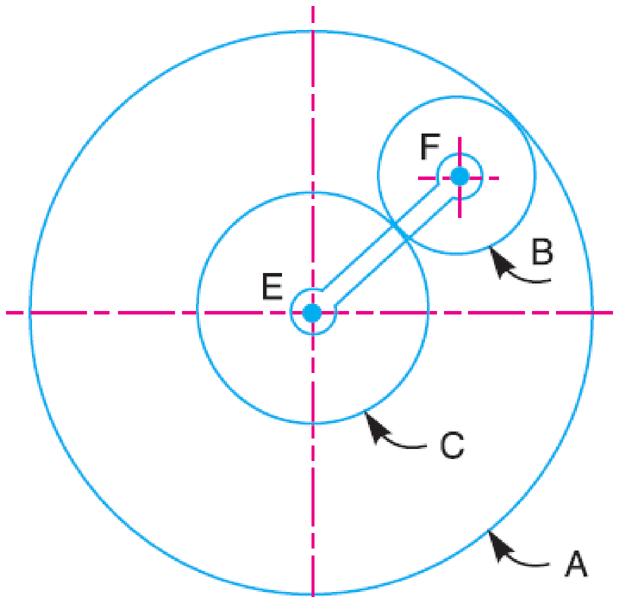

It consists of two co-axial shafts $S_1$ *and* $S_2$*, an annulus gear A which* is fixed, the compound gear (or planet gear) *B-C, the sun gear D* and the arm *H.* The annulus gear *A* meshes with the gear *B and the* sun gear *D meshes with the gear C.*

| Step No. | Conditions of motion | Revolutions of elements | | | |
|---|---|---|---|---|---|
| | | Arm | Gear D | Compound gear B-C | Gear A |
| 1. | Arm fixed-gear $D$ rotates through + 1 revolution | 0 | $+1$ | $-\frac{T_D}{T_C}$ | $-\frac{T_D}{T_C} \times \frac{T_B}{T_A}$ |
| 2. | Arm fixed-gear $D$ rotates through + $x$ revolutions | 0 | $+x$ | $-x \times \frac{T_D}{T_C}$ | $-x \times \frac{T_D}{T_C} \times \frac{T_B}{T_A}$ |
| 3. | Add + $y$ revolutions to all elements | $+y$ | $+y$ | $+y$ | $+y$ |
| 4. | Total motion | $+y$ | $x+y$ | $y - x \times \frac{T_D}{T_C}$ | $y - x \times \frac{T_D}{T_C} \times \frac{T_B}{T_A}$ |



# PRB I

*An epicyclic gear consists of three gears A, B and C as shown.*
*The gear A has 72 internal teeth and gear C has 32 external teeth. The gear B meshes with both A and C and is carried on an arm EF which rotates about the centre of A at 18 r.p.m.. If the gear A is fixed, determine the speed of gears B and C.*

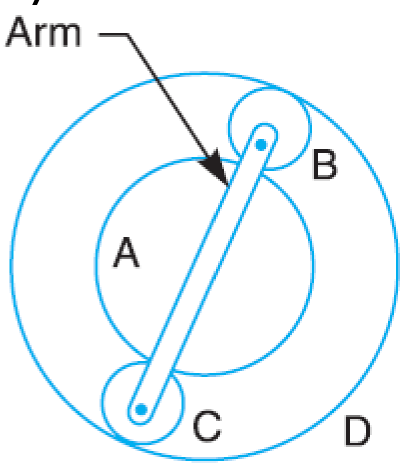


## PRB 2

An epicyclic train of gears is arranged as shown. How many revolutions does the arm, to which the pinions B and C are attached, make :

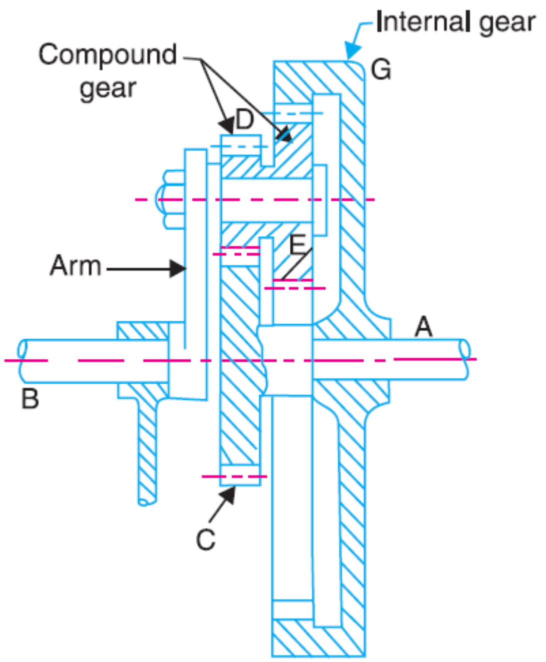
1. when A makes one revolution clockwise and D makes half a revolution anticlockwise, and
2. when A makes one revolution clockwise and D is stationary ?

The number of teeth on the gears A and D are 40 and 90 respectively. Gear B and C are identical.



# PRB 3

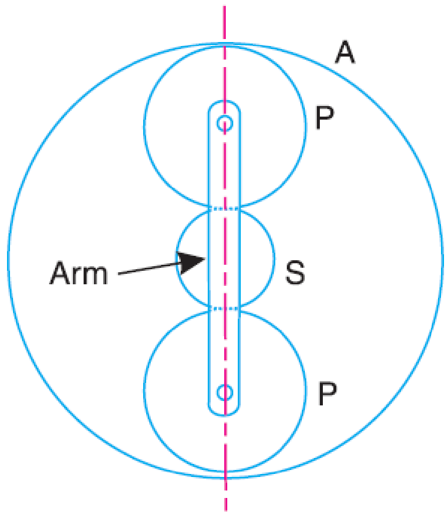
Two shafts A and B are co-axial. A gear C (50 teeth) is rigidly mounted on shaft A. A compound gear D-E gears with C and an internal gear G. D has 20 teeth and gears with C and E has 35 teeth and gears with an internal gear G. The gear G is fixed and is concentric with the shaft axis. The compound gear D-E is mounted on a pin which projects from an arm keyed to the shaft B. Sketch the arrangement and find the number of teeth on internal gear G assuming that all gears have the same module. If the shaft A rotates at 110 r.p.m., find the speed of shaft B.



# PRB 4

An epicyclic gear train, as shown in Fig. 13.37, has a sun wheel *S* of 30 teeth and two planet wheels *P*-*P* of 50 teeth. The planet wheels mesh with the internal teeth of a fixed annulus *A*. The driving shaft carrying the sunwheel, transmits 4 kW at 300 r.p.m. The driven shaft is connected to an arm which carries the planet wheels. Determine the speed of the driven shaft and the torque transmitted, if the overall efficiency is 95%.

**[Ans. 56.3 r.p.m. ; 644.5 N-m]**



An epicyclic reduction gear, as shown in Fig. 13.38, has a shaft *A* fixed to arm *B*. The arm *B* has a pin fixed to its outer end and two gears *C* and *E* which are rigidly fixed, revolve on this pin. Gear *C* meshes with annular wheel *D* and gear *E* with pinion *F*. *G* is the driver pulley and *D* is kept stationary.

The number of teeth are : $D = 80$ ; $C = 10$ ; $E = 24$ and $F = 18$.

If the pulley *G* runs at 200 r.p.m. ; find the speed of shaft *A*.

**[Ans. 17.14 r.p.m. in the same direction as that of *G*]**